

Everton Batista da Rocha  
Izabela Regina Cardoso de Oliveira

**Exercício 3.5-**

**Item a)**

*with(student) :*

*with(Student[Calculus1]) :*

*with(MTM) :*

$$w1 := u \rightarrow (2 \cdot \text{Pi})^{-1} \cdot \exp\left(-\frac{u^2}{2}\right) : w1(u);$$

$$\frac{1}{2} \frac{e^{-\frac{1}{2} u^2}}{\pi} \quad (1)$$

$$w2 := v \rightarrow (2 \cdot \text{Pi})^{-1} \cdot \exp\left(-\frac{v^2}{2}\right) : w2(v);$$

$$\frac{1}{2} \frac{e^{-\frac{1}{2} v^2}}{\pi} \quad (2)$$

$$\text{rho} := u \rightarrow 1 : \text{rho}(u);$$

$$1 \quad (3)$$

$$F := \text{Doubleint}(w1(u) \cdot w2(v) \cdot \text{rho}(u), u, v);$$

$$\int \int \frac{1}{4} \frac{e^{-\frac{1}{2} u^2} e^{-\frac{1}{2} v^2}}{\pi^2} du dv \quad (4)$$

$$\text{value}(F);$$

$$\frac{1}{8} \frac{\text{erf}\left(\frac{1}{2} \sqrt{2} u\right) \text{erf}\left(\frac{1}{2} \sqrt{2} v\right)}{\pi} \quad (5)$$

em que erf assume a seguinte expressão.

$$\frac{2 \cdot (\text{Integrate}(\exp(-t^2), t=0..x))}{\text{sqrt}(\text{Pi})}$$

$$\frac{2 \left( \int_0^x e^{-t^2} dt \right)}{\sqrt{\pi}} \quad (6)$$

**Item b)**

*with(student) :*

*with(Student[Calculus1]) :*

*with(MTM) :*

$$w1 := u \rightarrow (2 \cdot \text{Pi})^{-1} \cdot \exp\left(-\frac{u^2}{2}\right) : w1(u);$$

$$\frac{1}{2} \frac{e^{-\frac{1}{2} u^2}}{\pi} \quad (7)$$

$$w2 := v \rightarrow (2 \cdot \text{Pi})^{-1} \cdot \exp\left(-\frac{v^2}{2}\right) : w2(v);$$

$$\frac{1}{2} \frac{e^{-\frac{1}{2} v^2}}{\pi} \quad (8)$$

$$\text{rho} := u \rightarrow 1 - \frac{3}{2} \cdot \left(\frac{u}{\text{phi}}\right) + \frac{1}{2} \cdot \left(\frac{u}{\text{phi}}\right)^3 : \text{rho}(u);$$

$$1 - \frac{3}{2} \frac{u}{\phi} + \frac{1}{2} \frac{u^3}{\phi^3} \quad (9)$$

$$F := \text{Doubleint}(w1(u) \cdot w2(v) \cdot \text{rho}(u), u, v);$$

$$\int \int \frac{1}{4} \frac{e^{-\frac{1}{2} u^2} e^{-\frac{1}{2} v^2} \left(1 - \frac{3}{2} \frac{u}{\phi} + \frac{1}{2} \frac{u^3}{\phi^3}\right)}{\pi^2} du dv \quad (10)$$

$$\text{value}(F);$$

$$\frac{1}{8} \frac{1}{\pi^{3/2}} \left( \left( \frac{1}{2} \sqrt{\pi} \sqrt{2} \operatorname{erf}\left(\frac{1}{2} \sqrt{2} u\right) + \frac{3}{2} \frac{e^{-\frac{1}{2} u^2}}{\phi} \right. \right. \\ \left. \left. + \frac{1}{2} \frac{-e^{-\frac{1}{2} u^2} u^2 - 2 e^{-\frac{1}{2} u^2}}{\phi^3} \right) \sqrt{2} \operatorname{erf}\left(\frac{1}{2} \sqrt{2} v\right) \right) \\ \frac{1}{8} \frac{\operatorname{erf}\left(\frac{1}{2} \sqrt{2} u\right) \operatorname{erf}\left(\frac{1}{2} \sqrt{2} v\right)}{\pi} \quad (12)$$

em que erf assume a seguinte expressão.

$$\frac{2 \cdot (\text{Integrate}(\exp(-t^2), t=0..x))}{\text{sqrt}(\text{Pi})} \\ \frac{2 \left( \int_0^x e^{-t^2} dt \right)}{\sqrt{\pi}} \quad (13)$$