Review of *An autoregressive approach to spatio-temporal disease mapping* by Martinez-Beneito et al.

The authors propose a Bayesian method for risk estimation in space and time for lattice-based data. Although there are a number of models for space-time data, we still do not have a probability distribution class that combines flexibility and simplicity. The authors' model is a good candidate for this role. It is easily implemented (in WINBUGS and other software) and it can deal with temporal patterns that are spatially heterogeneous.

However, I think that there are some errors in the paper that require correction. The authors do not state explicitly if their successive CAR vectors $\boldsymbol{\phi}_{1:I,j}$ in equations (2.2) are independent in time. I assume they are because this property is used implicitly in the calculation of $\mathrm{Cov}(\boldsymbol{r}_j, \boldsymbol{r}_{j+1}|\ldots)$ in page 8, between lines 40 and 50 when all the cross-covariance $\mathrm{Cov}(\boldsymbol{\phi}_{1:I,j}, \boldsymbol{\phi}_{1:I,j+k}|\ldots)$ are canceled if $k > 1$.

If the successive CAR vectors

$$\boldsymbol{\phi}_{1:I,1}, \boldsymbol{\phi}_{1:I,2}, \ldots, \boldsymbol{\phi}_{1:I,J}$$

are i.i.d. and are distributed according to a CAR model with scale parameter σ^2_ϕ, then the proposed model (2.2) is a MA type rather than an AR type and this brings some consequences to the model the authors propose.

To be more specific, the classical MA model in time series is: $y_t = \epsilon_t + \theta\ \epsilon_{t-1}$ with the ϵ_t i.i.d with mean zero normal distribution. The AR model is

$$y_t = \phi\ y_{t-1} + \epsilon_t = \sum_{i=t}^{\infty} \phi^{t-k}\ \epsilon_{t-k}$$

As a consequence, $Cov(y_t, y_{t-k}) = 0$ for $k > 1$ in the MA case but it is $\phi^{t-k}\ \sigma^2_\epsilon \neq 0$ in the autoregressive case.

Returning to the authors' model, let us consider $\mu =$ and $\alpha_j = 0$ for all j. Additionally, let us also make $\theta_{ij} = 0$ for all i and all j. To simplify notation, let $\boldsymbol{\phi}_{1:I,j}$ be denoted simply by $\boldsymbol{\phi}_j$. Therefore, for $j > 1$, we have

$$\boldsymbol{r}_j = \boldsymbol{\phi}_j + \rho\, \boldsymbol{\phi}_{j-1} \tag{1}$$

If the vectors $\boldsymbol{\phi}_j$ are independent, this is simply a MA model. If it was meant to be an AR model it should be stated as

$$\boldsymbol{r}_j = \boldsymbol{\phi}_j + \rho\, \boldsymbol{r}_{j-1} \tag{2}$$

The major consequence is that the joint distribution stated in equation (3.1) is not correct and the model is not separable. This is not a bad consequence at all since separable models would have a hard time capturing the behavior generated in the authors' simulations. Perhaps that's the reason the model worked so well in that example.

Another important consequence is that, if each $\boldsymbol{\phi}_{1:I,j}$ is a Gaussian Markov random field (GMRF) with respect to the same neighborhood graph (this is assumed in the paper), then the resulting $\boldsymbol{r}_j$ in (1) is also a GMRF at time t *w.r.t. the same neighborhood graph*. This is not true for the AR model (2).

I have a few other minor comments that could strengthen the paper.

1. State in the Abstract that the model can be easily implemented in WinBUGS since this is a very important advantage of your proposal.

2. I am not a native speaker in English but I felt that the text needs a revision by someone who is completely proficient in that language. A few examples to illustrate the problems:

   - in the title, is *spatio*-temporal usual in English?
   - line 43 of page 1: ...and *from* spatial modelling
   - *bayesian* rather than Bayesian.
   - something such as *with the same level of agreement of* seems to be missing from line 35 of page 3.
   - line 54, page 4: takes profit seems awkward in English.
   - *On the other hand* is used many times in the paper without the complement *On one hand*.

3. Page 3, lines 23-25: It is confusing: what is desirable, the transfer or avoiding the transfer?

4. page 3, line 40: *spatial evolution at a single time* seems awkward. Evolution implies changes in time not at a single time.

5. Page 5, line 42: another possible extension of the model is to allow the parameter *rho* to be different for the two components, one ρ_θ for the non-spatial component, and another ρ_ϕ for the spatial component.

6. Page 5, line 50: The index of α should be $1:J$ rather than $1:I$.

7. Emphasize on page 5, after equations (2.2) that the vectors $\boldsymbol{\phi}_{1:I,j}$ are independent in time.

8. Page 6, line 38-41. This implies that α_j follows a random walk in time.

9. Page 7, lines 15 to 22: I could not understand this part.

10. Page 13, line 38: a *constant surface* means simply a constant or a surface that does not vary in time?

11. Figure 2 is very nice. You could improve the comparisons between the plots if they have the same vertical scale.

12. I could not understand the statement in page 21, lines 10-15.

13. Revise the paper with an English native speaker.