Comparing Area Yield Insurance with Farm Yield Insurance

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Two types of contracts

Notation

- **Farm level:** indemnity payments are triggered by losses measured at the farm level.
- A payment is made to the farmer if its yield (per planted hectare) is below a threshold.
- **Area level:** indemnity payments are triggered by losses measured at an area (county) level.
- A payment is made to the farmer if the area yield (per planted hectare) is below a threshold.

How to compare?

Equating to compare

- Which contract is better to the farmer?
- It depends on:
  - indemnity values,
  - thresholds,
  - premiums, etc
- We establish conditions to make a fair comparison between the two contracts.

Statistical framework

Notation

- Assume a region (a county) has n productive units
- All units has the same planted area.
- $S_1 = X_1$ is the yield of a single area with c.d.f F_1.
- $S_n = X_1 + \ldots + X_n$ is the total area yield.
- S_n/n is the average yield per unit with c.d.f. F_n.

Statistical framework

Insurer payment

- The insurer pays a quantity B to every policyholder in the group if $\dfrac{X(n)}{n} \leq F_{(n)}^{-1}(\alpha)$.
- The insurer payment is given by

$$\text{Insurer Pay} = \begin{cases} 0 & \text{if } \dfrac{S_n}{n} > F_n^{-1}(\alpha), \\ nB & \text{if } \dfrac{S_n}{n} \leq F_n^{-1}(\alpha) \end{cases}$$

Statistical framework

Loss for area yield contract

- Pure premium: defined to make the insurer expected loss equal to zero
- Expected payment by insurer is given by

$$nB \times P\left(\frac{S_n}{n} \leq F_n^{-1}(\alpha)\right) = nB\alpha$$

- Each unit pays $B\alpha$ as a pure premium.

Insurer Loss

Farm yield contract

- Insurer expected loss is given by

$$L_n = \begin{cases} 0 - nB\alpha & \text{if } \dfrac{S_n}{n} > F_n^{-1}(\alpha) \\ \\ nB - nB\alpha & \text{if } \dfrac{S_n}{n} \leq F_n^{-1}(\alpha) \end{cases}$$

- Since $P\left(S_n/n \leq F_n^{-1}(\alpha)\right) = \alpha$, we have that
  - $E[L_n] = -nb\alpha(1-\alpha) + nB(1-\alpha)\alpha = 0.$
  - $\text{Var}(L(n)) = (nB)^2\alpha(1-\alpha)$

Farm yield

Loss for farm yield contract

- Insurer loss

$$L_1 = \begin{cases} 0 - B\alpha & \text{if} \quad X_1 > F_1^{-1}(\alpha) \\ \\ B - B\alpha & \text{if} \quad X_1 \le F_1^{-1}(\alpha) \end{cases}$$

- with $E[L_1] = -B\alpha(1-\alpha) + B(1-\alpha)\alpha = 0$
- and $\text{Var}(L_1) = B^2\alpha(1-\alpha)$.

Comparing contracts by moments

Which one is better?

- For whom?
- One area yield contract with n units and a set of n individual contracts.
- A fair way to compare:

$$\frac{\mathrm{Var}(L_n)}{\mathrm{Var}(nL_1)} = \frac{(nB)^2\alpha(1-\alpha)}{n^2[B^2\alpha(1-\alpha)]} = 1$$

- No differences if we base the comparison on the first two moments.
- Which other criteria? Look ate the probabilities of "wrong" actions.

Inefficiency

Definition

- Under area yield, two inefficient actions
  - to pay indemnity to a farmer when he had no loss.
  - to not pay a farmer when he had a loss.
- This happens with probabilities

$$P\left(X_i \leq F_1^{-1}(\alpha) \text{ and } \frac{S_n}{n} \geq F_n^{-1}(\alpha)\right) = P(\text{nao pagar quando devia})$$

$$P\left(X_i \geq F_1^{-1}(\alpha) \text{ and } \frac{S_n}{n} \leq F_n^{-1}(\alpha)\right) = P(\text{pagar quando não devia})$$

- Under the farm yield contract no inefficiency.

Calculating probabilities

Which is larger?

- X_i=crop yield for farm i, and $S_n = \sum_i X_i$.
- $(X_1, ..., X_n) \sim N_n(M, \Sigma)$, where $M = \{\mu, ..., \mu\}$ and
- $$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \cdots & \rho & \rho \\ \rho & 1 & & & \rho \\ \vdots & & \ddots & & \vdots \\ \rho & & & 1 & \rho \\ \rho & \rho & \cdots & \rho & 1 \end{pmatrix}$$
- Let $W = X_1$ and $Y = X_2 + ... + X_n$
- We want $P\left(W < a \text{ and } \frac{W+Y}{n} > b\right)$ and $P\left(W > a \text{ and } \frac{W+Y}{n} \leq b\right)$

Crunching many numbers later

Result

- We find that

$$P\left(W < a \ \text{ and } \ \frac{W+Y}{n} > b\right) = P\left(W > a \ \text{ and } \ \frac{W+Y}{n} \leq b\right)$$

- These probabilities are equal to

$$\alpha\left[1 - E_V\left[\Phi\left(c\left(V(1+\rho(n-1)) - \frac{\sqrt{n}(a-\mu)}{\sigma}\right)\right)\right]\right]$$

- where $c = -1/\sqrt{(n-1)(1+(n-2)\rho) - (n-1)\rho^2}$,
- $\alpha = \Phi\left(\frac{a-\mu}{\sigma}\right)$
- and V is a truncated standard normal random variable, where the truncation point is $\frac{a-\mu}{\sigma}$.

Decision Making

Utility function $U(w)$

- w is the wealth of an agent.
- $U'(w) > 0$
- U(w) crescente. Isto significa que quanto mais riqueza, melhor.
- $U"(w) < 0$
- O incremento de $U(w)$ diminui medida que se enriquece.
- Agent faces a random loss X ending with $w - X$.
- Rational agent takes decision which maximizes expected utility.

Choices

Final wealth

- Farm yield contract: $W_f = w - X + BI[X < F^{-1}(\alpha)] - B\alpha$
- Area yield contract: $W_a = w - X + BI[\overline{X}_n < F_n^{-1}(\alpha)] - B\alpha$
- We have $E(W_f) = E(W_a)$ but $\text{Var}(W_f) < \text{Var}(W_a)$
- by the properties of utility function we have $E(U(W_f)) > E(U(W_a))$
- Hence, the area based contract is better for the insured.