Analysis of Rainfall Derivatives Using Daily Precipitation Models: Opportunities and Pitfalls

Martin Odening, Oliver Musshoff, and Wei Xu

Abstract

This study examines rainfall variability and its implications for wheat production risk in northeast Germany. The hedging effectiveness of rainfall options and the role of geographical basis risk are analyzed using a daily precipitation model. Simpler pricing methods such as the burn analysis and the index value simulation serve as benchmarks for comparison. It is found that the choice of statistical approach may lead to considerable differences in the estimation results. Daily precipitation models should be used with some caution in the context of derivative pricing because they tend to underestimate rainfall variability. This is unexpected, because daily simulation models are usually preferred in the context of temperature-based weather indexes.

Key words: hedging effectiveness, precipitation modeling, weather derivatives

Weather, though an important production factor in agriculture, can hardly be controlled. In fact, weather risks are a major source of uncertainty in crop production. Traditionally, producers have tried to compensate for the negative economic consequences of bad weather events by purchasing insurance. However, weather derivatives, a new class of financial instruments that permit the trade of weather-related risks, emerged in the mid-1990s. These instruments include futures, options, and swaps, all of which are traded over the counter and on formal exchanges such as the Chicago Mercantile Exchange (CME). Common underlying weather variables are temperature, rainfall, or wind.

The advantage of weather derivatives is that their payoffs are determined in a transparent manner and with low transaction costs. Moreover, they are not affected by moral hazard or adverse selection, which can be serious problems for insurance companies. Yet, considerable risk may remain with the producer when using weather derivatives (*a*) because individual yield variations are not, in general, completely correlated with the relevant weather variable, and (*b*) because of geographical basis risk—i.e., the difference between the weather index at a reference point and at a specific farm location.

To date, it has been unclear if weather derivatives would permeate agriculture. But the literature increasingly deals with the question of whether weather derivatives can also play a role as agribusiness risk management tools (e.g.,

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Turvey, 2001, 2005; Richards, Manfredo, and Sanders, 2004; van Asseldonk and Oude Lansink, 2003).

Analyzing the hedging effectiveness of weather derivatives requires three interrelated problem areas to be solved: first, the statistical modeling of relevant weather variables; second, quantifying the relationship between weather variables and production; and third, developing a theoretically consistent pricing model. This paper focuses on the availability of a reliable statistical model of the weather variable, which is important because it facilitates the quantification and prediction of the weather risk. Moreover, it is a necessary ingredient for calculating the price of the weather derivative, i.e., insurance costs. Substantial research exists with regard to the statistical modeling of temperature-related derivatives (e.g., Campbell and Diebold, 2005; Jewson and Brix, 2005), but few papers deal with statistical models of rainfall derivatives, despite the importance of rainfall for agricultural production.

Obviously there are significant differences between analyzing rainfall and temperature. Rainfall is a binary event and evolves much more erratically than temperature changes. Further, the correlation between rainfall amounts at adjacent locations is relatively low. Most of the existing applications of rainfall insurance prefer to estimate the probability law of the rainfall index directly, assuming an appropriate distribution function (Stoppa and Hess, 2003; Skees et al., 2001). Turvey (1999) compares the results of an empirical and a normal distribution. Cao, Li, and Wei (2004) were the first to suggest calculating the derivative payoff from a rather subtle stochastic process of daily rainfall.

This paper has two objectives. The first is the development, estimation, and comparison of different precipitation models. Comparing these models reveals their strengths and weaknesses and facilitates the assessment of their usefulness for analyzing rainfall-based weather derivatives or insurance in agriculture. In particular, we wish to investigate whether the use of daily simulation models can actually improve the pricing of rainfall derivatives, as is the case for temperature-related derivatives. The second objective is to examine the role of basis risk for the hedging effectiveness of rainfall derivatives. Our focus lies on the quantification of geographical basis risk by means of a de-correlation analysis. The precipitation model and the de-correlation analysis are applied to a case study that considers wheat production in Germany.

The remainder of the paper is structured as follows. First, the theoretical background of weather derivatives is briefly reviewed. Next, alternatives of index modeling are discussed. A daily precipitation model is then presented, followed by an empirical application. Using rainfall data from the Brandenburg region of Germany, put options on two rainfall indexes are priced with different methods, and the effect of wheat producers' exposure to risk is examined. The paper ends with conclusions related to the proposed statistical approach.

Valuation of Weather Derivatives

Pricing Weather Derivatives

Financial theory asserts that the price of a contingent claim, F, which depends on stochastic variable I and expires at time T, can be calculated according to (Neftci, 1996, p. 297):

(1) $F = \tilde{E}[W(I)] \cdot \exp(-r \cdot T)$,

where $W(I)$ denotes the payoff of the derivative at expiration, and r is the risk-free interest rate. The variable E represents expectation, conditional on the information available at present, and the tilde (~) indicates that the expectation of the derivative payoff is calculated by means of risk-neutral probabilities instead of real-world probabilities. The use of risk-neutral probabilities ensures the derivative price is arbitrage free. This is a desirable property when the derivative is traded before it expires.

To illustrate this concept, consider variable I, which follows a geometric Brownian motion with an expected growth rate α and volatility σ. To calculate the price of a derivative in a risk-neutral world, the actual growth rate α must be reduced by a risk premium $(\lambda * \sigma)$ while volatility remains unchanged. The variable λ is known as the market price of risk for I. Hull (2006, p. 590) proves that if I is a traded asset, the market price of risk is given by $\lambda = (\alpha - r)/\sigma$. Inserting this into the above expression for the risk premium shows that the risk-adjusted discount rate is simply the risk-free interest rate r. This fact is well known from the Black-Scholes model.

However, the direct application of no-arbitrage models to weather derivatives is impractical since weather cannot be traded. If I is a weather index, it is not possible to construct a risk-free hedge portfolio consisting of I and the derivative F, and hence the price of the derivative must account for the market price of weather risk. Weather derivatives are typical examples of an incomplete market. Unfortunately, there is no unique way to "risk-neutralize" the objective (real-world) probability distribution of the underlying I in the case of an incomplete market. Accordingly, many arbitrage-free prices for the derivative exist (Benth, 2004, p. 88).

Various proposals for treating this problem can be found in the literature. Alaton, Djehiche, and Stillberger (2002) determine the market price for weather risk to be an implicit parameter such that the theoretical pricing model matches the observable market prices for some contracts. Of course, this approach is only practical if a market already exists for weather derivatives. Cao and Wei (2003) and Richards, Manfredo, and Sanders (2004) apply an extended version of Lucas' (1978) equilibrium pricing model where direct estimation of the market price of weather risk is avoided. Instead, pricing is based on the stochastic processes of the weather index and an aggregated dividend. Moreover, an assumption about the utility function of a representative investor is required.

Turvey (2005) refers to the capital asset pricing model in order to estimate the market price of risk. From the CAPM we have:

$$(2) \quad \mu = r + (\mu_M - r) * \rho * \frac{\sigma}{\sigma_M},$$

where μ and σ are the expected value and the standard deviation of the returns of an asset, respectively. Variables μ_M and σ_M denote the corresponding values of the market portfolio, and ρ measures the correlation between the asset and the market portfolio. Combining (2) with the definition $\lambda = (\mu - r)/\sigma$ provides an estimable relation for λ:

$$(3) \quad \lambda = \rho * \frac{(\mu_M - r)}{\sigma_M}.$$

In the subsequent application, we will argue in accordance with Hull (2006, p. 552) that rainfall indexes have no (or a negligible) correlation with stock market returns, i.e., rainfall variability is not a systematic risk. In that case, the market price of risk is zero, and no correction with the distribution of the weather index is necessary. This means the expectation in (1) can be calculated with real-world probabilities.

Determining Hedging Effectiveness

Agricultural producers rarely evaluate a weather derivative via its contribution to a well-diversified investment portfolio. Rather, they are interested in knowing if and to what extent the existing yield risk can be eliminated by holding this security. The risk reduction that can be attributed to weather insurance is measured by comparing the revenue distribution of a production activity or a whole farm with and without having the weather derivative. The present value of revenues R of a farmer who produces output Q and holds a position of the weather derivative is defined by:

(4) $R = \left(Q(I) * P + W(I)\right) * \exp(-r * T) - F,$

where P is the product price, which is assumed to be constant for the sake of simplicity. Without insurance, the terms W and F vanish. Output Q is a function of the stochastic weather index I and other controllable or stochastic factors. Obviously, the risk-reducing potential of any weather insurance depends on the correlation between the weather index and the considered agricultural product. Further on, this relationship will be captured by the estimation of a "production function."

Analyzing the hedging effectiveness of risk management tools is usually carried out in an expected utility framework. A widely used utility function, which we also apply here, is the negative exponential:

(5) $U(R) = -\exp(-\lambda_a * R),$

where λ_a is the absolute risk-aversion parameter. The effect of weather insurance can then be expressed in terms of certainty equivalence (CE):

(6) $CE = E^{-1}\left(U(R)\right).$

Statistical Modeling of Weather Indexes

The previous section made clear that the probability distribution of the weather index I at the time of expiration is crucial to the assessment of any weather derivative. The distribution of the weather index influences the revenue distribution through the production function and the derivative payoff and also determines the cost of insurance, i.e., the derivative price.

From a statistical viewpoint, there are three alternatives with regard to the modeling of weather risk. On the one hand, the weather index distribution (e.g., cumulative rainfall in May) can be estimated directly, either parametrically or nonparametrically. On the other hand, the relevant weather index can be derived from a daily model of a generic weather variable (e.g., daily rainfall) through appropriate aggregation.

The corresponding approaches are the burn analysis (also referred to as burn rate method), the index value simulation, and the daily simulation. These modeling approaches are briefly described in turn. For a more thorough discussion, interested readers are referred to Jewson and Brix (2005, chapters 3, 4, and 6).

Burn Analysis

In a nonparametric burn analysis, the general pricing formula (1) is implemented in a simple manner:

(7) $F = \exp(-r * T) * \left[\frac{1}{n} * \sum_{t=1}^{n} W(I_t)\right].$

Calculating (7) involves the following steps. First, weather data over a time horizon of n years are collected (and cleansed, if necessary). Next, the index value and the hypothetical payoffs of the derivative are determined for each year in the sample period. Finally, the payoff average is calculated and discounted with the risk-free interest rate r. This means that the burn analysis utilizes the empirical distribution of the rainfall index. No further statistical model is required.

Although this method is widely practiced, it has been criticized in the literature. Turvey (2005) states that a burn analysis is only backward-looking, and implicitly assumes that historical patterns will repeat themselves, while Cao, Li, and Wei (2003) report that derivative prices produced by this method are rather sensitive to the number of observations. Moreover, Jewson and Brix (2005) emphasize that the burn analysis has difficulties predicting the occurrence of extreme weather events.

Index Value Simulation

An index value simulation follows steps similar to the burn analysis, but the empirical distribution is replaced by a statistical model for the weather index or the derivative payoff. This can be a nonparametric distribution, for example a kernel density, or a parametric distribution. The choice of the functional form of the distribution is usually based on theoretical considerations (e.g., a nonnegative domain for rainfall amounts) and is supported by goodness-of-fit tests.

Parameters of the distribution can be estimated from historical data with standard methods such as the method of moments or maximum likelihood. With an appropriate distribution, at-hand values for the precipitation index are randomly drawn and the discounted payoff of the derivative is determined. The derivative price is again obtained by calculating the average discounted payoff.

If the correct distribution is known and the parameters can be estimated precisely, then the index value simulation will produce more accurate results than the burn analysis, because random errors can be eliminated by choosing a sufficiently high number of random draws. However, in contrast to the modeling of financial variables, there is little theoretical guidance for modeling the distributions of weather indexes, and hence the danger exists of misspecifying the model.

Daily Simulation

Instead of modeling the weather index or the payoff distribution directly, one can alternatively develop a statistical model for the stochastic process of the underlying weather variable (daily rainfall or average daily temperature). Such a model describes the dynamics of the weather variable over time and can be used for simulation. The weather index can be derived from the simulated sample path by summing up daily precipitation or daily average temperature, respectively, in the relevant accumulation period. The subsequent steps for calculating the derivative price are identical to the index value simulation.

This procedure is initially more complex, yet potentially favorable for several reasons. First, the ways in which daily models can be used are very flexible, because practically all yield-relevant events such as the sums of precipitation or temperature for different accumulation periods, dry spells, or extreme precipitation can be determined for any time period. In contrast, directly estimating the weather index distribution is usually only valid for a particular index. Second—and this seems even more important than the higher flexibility—the accuracy of daily-based models is higher due to a considerably larger number of observed values (Brix, Jewson, and Ziehmann, 2002). If observations from N years are available to estimate the parameters of a weather index distribution, then $365 \times N$ observations can be used to estimate the stochastic process parameters of the underlying daily weather variable. Finally, daily simulation permits incorporating weather forecasts into the pricing model.

Presumably for these reasons, pricing and analyzing the effectiveness of temperature-related derivatives mainly originate from daily temperature models. Despite the aforementioned differences between the stochastic processes of temperature and rainfall, it seems promising to apply the daily modeling approach in the context of rainfall-based insurance. We pursue this idea in the next section.

A Daily Precipitation Model

A precipitation model should be able to capture the following characteristics of daily rainfall. First, the probability of rainfall occurrence obeys a seasonal pattern.

Rainfall in Europe, for example, is more likely in winter than in summer. Second, the sequence of wet and dry days follows an autoregressive process. This means the probability of a rainy day is higher if the previous day was wet. Third, the amount of precipitation on a wet day varies with the season. Rainfall in Europe is more intensive in summer than in winter. Fourth, the volatility of the amount of rainfall also changes seasonally. In Europe, it is higher in summer than in winter.

In the following, a daily precipitation model is described which can depict these characteristics. According to Moreno (2002) and Cao, Li, and Wei (2004), the stochastic process of daily precipitation can be decomposed into two parts: (*a*) a stochastic process for the binary event "rainfall" and "dryness," and (*b*) a distribution for the amount of precipitation given a rainy day. In order to specify the first component of the model, we define the random variable X_t:

$$(8) \quad X_t = \begin{cases} 0 & \text{if day } t \text{ is dry,} \\ 1 & \text{if day } t \text{ is rainy.} \end{cases}$$

It is assumed that X_t follows a first-order Markov chain. The probability (p_t) that it will rain on day t can be calculated as:

$$(9) \quad p_t = p_{t-1} \cdot q_t^{11} + (1 - p_{t-1}) \cdot q_t^{01}, \quad t = 1, 2, \ldots, 365,$$

where q_t^{01} describes the transitional probability of rain on day t and dryness on the previous day $t-1$. Analogously, q_t^{11} represents the transition probability between two successive rainy days. Note that the transition probabilities q_t^{01} and q_t^{11} vary with time.

The second part of the model, i.e., the precipitation amount y_t on day t, is represented by a sequence of continuous random variables with independent distributions. In the literature, various distributions with a nonnegative domain are discussed, including, among others, the exponential distribution and the gamma distribution (Woolhiser and Roldan, 1982). The mixed exponential distribution has proven to be especially flexible (Wilks and Wilby, 1999). The density function is given by:

$$(10) \quad f(y_t \mid X_t = 1) = \frac{\alpha_t}{\beta_t} \cdot \exp\left(\frac{-y_t}{\beta_t}\right) + \frac{1-\alpha_t}{\gamma_t} \cdot \exp\left(\frac{-y_t}{\gamma_t}\right),$$

$$\text{with } 0 \le \alpha_t \le 1 \text{ and } 0 < \beta_t < \gamma_t.$$

The mixed exponential is a weighted combination of two simple exponential distributions and inherits their properties. The advantage of this distribution is that it can better represent extreme events compared with a simple exponential (cf. Hansen and Mavromatis, 2001). The parameters of the mixed exponential distribution α_t, β_t, and γ_t are also time-varying, thereby taking into account the seasonality of precipitation.

In this form, however, the model is not estimable. In order to reduce the number of parameters to be estimated, each of the time-varying parameters is developed by a finite Fourier series:

$$(11) \quad \theta_{tj} = a_{j0} + \sum_{k=1}^{m_j} \left[a_{jk} \cdot \sin\left(\frac{t \cdot k}{\tau} - b_{jk}\right) \right],$$

where $\theta_{t1} = q_t^{01}$, $\theta_{t2} = q_t^{11}$, $\theta_{t3} = \alpha_t$, $\theta_{t4} = \beta_t$, $\theta_{t5} = \gamma_t$, and $\tau = 365/(2 \cdot \pi)$; a_{jk} and b_{jk} denote the Fourier coefficients; and m_j is the maximum number of harmonics needed to specify the seasonal cycles. The variable m_j can be determined by means of a model selection criterion, e.g., the Akaike information criterion (AIC). The Fourier coefficients for q_t^{01} and q_t^{11} are estimated by maximizing the following log-likelihood functions (Woolhiser and Pegram, 1979):

$$(12) \quad \ln L_1 = \sum_{t=1}^{365} \left[c_t^{00} \cdot \ln(1 - q_t^{01}) + c_t^{01} \cdot \ln(q_t^{01}) + c_t^{10} \cdot \ln(1 - q_t^{11}) + c_t^{11} \cdot \ln(q_t^{11}) \right],$$

where c_t^{ij} denotes the observed number of transitions from state i at day $t-1$ to state j at day t.

In order to estimate the Fourier coefficients for α_t, β_t, and γ_t, we maximize the following log-likelihood function (Woolhiser and Pegram, 1979):

$$(13)\quad \ln L_2 = \sum_{t=1}^{n} \ln\left[f(y_t | X_t = 1)\right],$$

where n denotes the number of rainy days in the sample.

Based on this model, Wilks (1999) provides a computational procedure for the simulation of sample paths of daily rainfall. First, three independent uniform random variables, $u_{1,t}$, $u_{2,t}$, and $u_{3,t}$, are generated, with $u_{1,t}, u_{2,t}, u_{3,t} \in [0, 1]$. The sequences of wet and dry days (8) can then be simulated by comparing the estimated probabilities of daily rainfall occurrence (9) and $u_{1,t}$ as follows:

$$(14)\quad X_t = \begin{cases} 0 & \text{if } u_{1,t} > p_t, \\ 1 & \text{otherwise.} \end{cases}$$

For a rainy day ($X_t = 1$), the rainfall amount y_t^{sim} can be simulated according to:

$$(15)\quad y_t^{sim} = \tilde{y} - \varpi_t \cdot \ln(u_{2,t}),$$

where $\tilde{y}$ is the minimal rainfall amount for a day to be recorded as rainy ($\tilde{y}$ equals 0.1 mm for weather stations in Germany). The parameter ϖ is chosen as:

$$(16)\quad \varpi_t = \begin{cases} \beta_t & \text{if } u_{3,t} \le \alpha_t, \\ \gamma_t & \text{otherwise.} \end{cases}$$

The sequence y_t^{sim}, $t = 1, 2, \ldots, 365$, facilitates determining one iteration of the considered rainfall index. The entire procedure is repeated 50,000 times.

Application: Valuation of Rainfall Options for Grain Producers in Northeast Germany

Grain production in northeast Germany, Brandenburg in particular, is highly affected by rainfall risk. During the relevant period of April to June over the last 20 years, average yearly rainfall in Brandenburg was between 64 mm and 258 mm of precipitation (with a mean of 151 mm), and the grain yields have fluctuated similarly. The correlation between rainfall and yields results from the sandy soil possessing little water-storing capacity, as well as the lack of irrigation. Currently there is no possibility of insuring against yield losses caused by low rainfall.

In view of the extreme crop failures in the drought years 2000 and 2003, during which time the government had to provide disaster relief in order to save farmers from insolvency, there is a pronounced interest in introducing some kind of rainfall insurance. By purchasing a put option on some rainfall index, a grain producer is (partially) insured against revenue losses due to little precipitation in the growing season. Prior to the analysis of the hedging effectiveness of a particular instrument, we investigated the properties of the historical simulation, the index value simulation, and the daily simulation for a variety of rainfall indexes.

Definition of the Rainfall Indexes

Two different types of precipitation indexes are defined: a cumulative rainfall index and a deficit index. The cumulative index (I^c) is the sum of daily rainfall amount in a certain accumulation period:

$$(17)\quad I^c = \sum_{t=1}^{x} y_t,$$

where x denotes the length of the accumulation period. We call this index the "rainfall sum." Existing empirical studies on rainfall insurance usually apply this type of index. As an alternative, we suggest a rainfall deficit index (I^d) defined as:

$$(18)\quad I^d = \sum_{\tau=1}^{z} \min\left(0, \sum_{t=(\tau-1)\cdot s+1}^{\tau \cdot s} y_t - y^{min}\right).$$

This index measures the shortfall of the rainfall sum in an s-days period relative

Table 1. Probability Distribution Functions Selected by Different Goodness-of-Fit Tests

	Rainfall Sum Index				Rainfall Deficit Index			
	— Accumulation Period —				— Accumulation Period —			
Goodness-of-Fit Test	**Jan–Dec**	**Mar–July**	**Apr–June**	**June**	**Jan–Dec**	**Mar–July**	**Apr–June**	**June**
Chi Square:	**Beta**	**Weibull**	**Beta**	**Pearson VI**	**Beta**	**Weibull**	**Beta**	**Beta**
• Test Value	3.90	4.41	7.30	11.96	6.50	6.84	10.28	9.00
• Critical Value*	16.92							
Kolmogorov-Smirnov:	**Weibull**	**Weibull**	**Gamma**	**Lognormal**	**Beta**	**Weibull**	**Beta**	**Beta**
• Test Value	0.08	0.07	0.07	0.08	0.05	0.06	0.12	0.07
• Critical Value*	0.87	0.87	1.36	1.36	1.36	0.87	1.36	1.36
Anderson-Darling:	**Weibull**	**Erland**	**Gamma**	**Lognormal**	**Beta**	**Weibull**	**Beta**	**Beta**
• Test Value	0.42	0.23	0.44	0.54	0.53	0.29	0.66	0.82
• Critical Value*	0.76	2.49	2.49	2.49	2.49	0.76	2.49	2.49

* Critical values at a 95% significance level.

to a reference level y^{min}. This shortfall is cumulated over z periods. Hence, the construction principle is quite similar to that of degree-day indexes, which are widely used for the specification of temperature derivatives.[1] The rainfall sum and the rainfall deficit are calculated for four accumulation periods: January 1–December 31, March 1–July 31, April 1–June 30, and June 1–June 30. The variable s is set to five days, and y^{min} equals the average five-day precipitation in the respective accumulation period.

Specification and Estimation of the Precipitation Models

Estimation of the rainfall index distributions is based on rainfall data measured in Berlin-Tempelhof from January 1, 1948 to December 31, 2005. This means $N = 58$ observations are available for estimating the empirical and the parametric distribution of the two rainfall indexes, and $n = 21{,}170$ observations for estimating the daily precipitation model. The most appropriate parametric distributions for the eight indexes are determined by means of three goodness-of-fit tests: the Chi-Square test, the Kolmogorov-Smirnov test, and the Anderson-Darling test.[2] That distribution which showed the smallest value of the respective test criterion was selected. Only distributions with a nonnegative domain for the rainfall sum and a nonpositive domain for the rainfall deficit were considered.

Table 1 displays the estimation results. For rainfall deficit, the beta distribution, in most cases, offers the best approximation to the empirical distribution. An exception is the accumulation period from March–July, where the Weibull distribution is more appropriate. The choice of a parametric distribution for the rainfall sum is more complicated, initially because more functions come into play, and additionally because the three statistical tests suggest different distributions for the same accumulation period. Considering this sensitivity, we did not select a single distribution for the index value simulation but ran two variants labeled "index value simulation I" (IVS I) and "index value simulation II" (IVS II) thereafter.

[1] This definition may appear unusual since the deficit index will take negative values. However, the definition is convenient for the present application because the relationship between the index and the production output is the same as for the rainfall sum.

[2] The calculations were carried out with BestFit. Detailed information on the properties and the assumptions of these goodness-of-fit tests can be found, for example, in D'Agostino and Stephens (1986) and Vose (2005, p. 240).

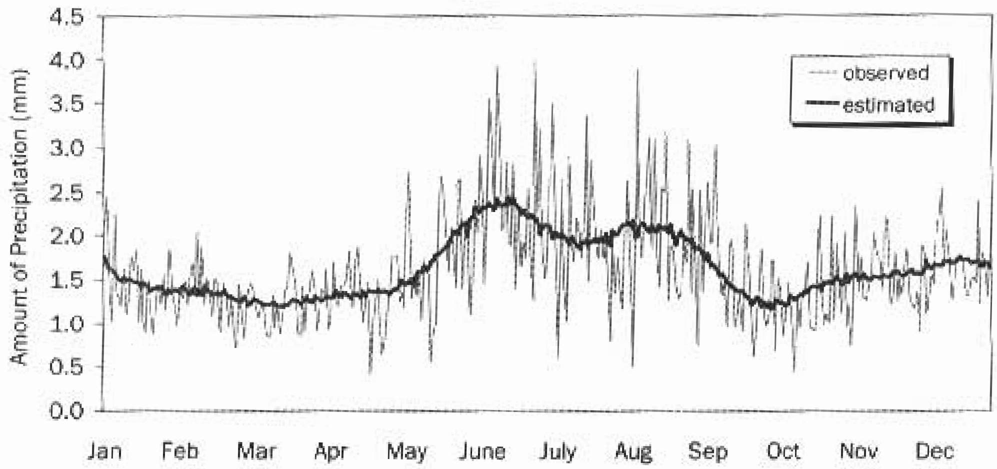

Figure 1. Observed and Estimated Average Daily Precipitation (weather station Berlin-Tempelhof)

To estimate the daily precipitation model parameters (8) to (11), the likelihood functions (12) and (13) were maximized with a genetic algorithm.[3] The assumption of a mixed exponential distribution for the daily rainfall amount is supported by a Kolmogorov-Smirnov test at a 95% significance level. The number of harmonics of the Fourier series, m_j, are determined using the Akaike information criterion (AIC). The AIC value is minimal for $m = 9$ in the case of the transition probabilities, and $m = 7$ in the case of the mixed exponential distribution parameters. Figure 1 shows the actual and the estimated daily rainfall over the course of a year. Figure A1 in the appendix depicts the corresponding transition probabilities q_t^{01} and q_t^{11}. Obviously, the model not only fits the yearly average, but also the seasonality of the rainfall amounts as well. From appendix Figure A2 it can be seen that the standard deviation of the estimated daily precipitation is actually higher in summer than in winter. Hence, the model reflects the aforementioned characteristics of daily rainfall.

A pitfall of the daily precipitation model is the underestimation of the cumulated rainfall variance over a period of several weeks. This underestimation of the variance has already been observed in a different context and has been termed "low frequency variability bias" (Hansen and Mavromatis, 2001; Dubrovsky, Buchtele, and Zalud, 2004). For example, the sample standard deviation of the cumulative precipitation from January–December is 95.09 mm, whereas the daily precipitation model only shows a value of 75.53 mm. One can expect that the daily simulation will also result in biased options prices because options prices are sensitive to volatility.

Hansen and Mavromatis (2001) discuss various methods for reducing the low frequency variability bias. In this study we take several measures. First, the transition probabilities q_t^{01} and q_t^{11} are estimated by their empirical sample counterparts, which clearly show a higher variability than those based on the Fourier series (see appendix Figure A1). Moreover, the parameters of the mixed exponential distribution α_t, β_t, and γ_t are determined in such a way that the resulting standard deviation exactly fit the sample standard deviation of the daily rainfall amounts shown in appendix Figure A2(b). Second,

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[3] Genetic algorithms are heuristic search procedures which are able to solve complex optimization problems by mimicking the optimization strategy of biological evolution. For a detailed exposition, see Goldberg (1989) or Mitchell (1996).

Table 2. Comparison of Different Precipitation Models

PANEL A. Moments of the Rainfall Index

	Rainfall Sum Index				Rainfall Deficit Index			
	— Accumulation Period —				— Accumulation Period —			
Precipitation Model	**Jan–Dec**	**Mar–July**	**Apr–June**	**June**	**Jan–Dec**	**Mar–July**	**Apr–June**	**June**
BA:								
‣ Mean	578.32	257.13	160.34	69.54	-257.99	-116.31	-75.88	-29.52
	(11.94)	(7.31)	(6.43)	(5.23)	(4.82)	(2.35)	(2.26)	(1.59)
‣ Standard Deviation	95.09	53.46	51.37	40.21	37.48	18.79	17.13	13.04
	(7.92)	(4.84)	(3.54)	(5.72)	(2.60)	(1.59)	(1.68)	(1.04)
IVS I:	**Beta**	**Weibull**	**Beta**	**Pearson VI**	**Beta**	**Weibull**	**Beta**	**Beta**
‣ Mean	578.32	254.27	160.34	75.54	-257.99	-116.31	-75.88	-29.52
	(12.48)	(8.29)	(6.66)	(7.43)	(5.26)	(2.54)	(2.28)	(1.88)
‣ Standard Deviation	95.09	60.79	51.37	64.00	37.48	18.86	17.13	13.04
	(6.30)	(5.70)	(3.37)	(16.36)	(2.32)	(1.74)	(1.37)	(1.04)
IVS II:	**Weibull**	**Erland**	**Gamma**	**Lognormal**	**Beta**	**Weibull**	**Beta**	**Beta**
‣ Mean	572.86	256.69	160.34	70.57				
	(13.70)	(7.72)	(6.34)	(5.74)		[see IVS I]		
‣ Standard Deviation	105.73	53.44	51.99	45.66				
	(10.57)	(5.58)	(5.93)	(9.95)				
DS I:								
‣ Mean	578.72	257.10	161.54	68.13	-236.62	-108.27	-70.45	-28.16
	(9.76)	(5.71)	(5.25)	(5.07)	(3.79)	(2.51)	(1.69)	(2.05)
‣ Standard Deviation	75.53	51.93	42.91	29.24	28.41	19.62	15.91	11.46
	(5.87)	(4.74)	(4.92)	(3.68)	(2.37)	(1.75)	(2.06)	(1.22)
DS II:								
‣ Mean	579.21	256.31	160.91	69.15	-246.66	-113.89	-74.36	-30.09
	(8.95)	(6.96)	(5.95)	(4.24)	(3.12)	(2.67)	(1.99)	(1.76)
‣ Standard Deviation	83.26	56.35	46.03	32.15	29.65	20.55	16.74	11.82
	(6.94)	(5.00)	(4.90)	(4.13)	(2.42)	(1.89)	(1.69)	(1.11)

Notes: BA = burn analysis, IVS = index value simulation, DS = daily simulation; mean and standard deviation values are in mm; values for the put and call options are in €; values in parentheses are standard errors.

following Dubrovsky, Buchtele, and Zalud (2004), a second-order Markov process (as opposed to a first-order) is estimated. Thus, longer sequences of consecutive rainy and dry days, respectively, may occur, which leads to a higher standard deviation of the cumulated precipitation. In what follows, the original and the modified daily precipitation model are called "daily simulation I" (DS I) and "daily simulation II" (DS II), respectively.[4]

[4] In addition to these two models, other model versions have been specified and estimated. For example, the mixed exponential distribution has been replaced by a kernel density estimator as suggested by Rajagopalan, Lall, and Tarboton (1996). The results, however, did not change significantly.

Results of the Precipitation Models

Panel A of Table 2 presents the mean and the standard deviation of the rainfall index distributions obtained by applying the burn analysis, the index value simulation, and the daily simulation.[5] The results of the parametric methods (index value simulation and daily simulation) are based on 50,000 repetitions. Consequently, sampling errors are virtually eliminated.

[5] Note that the mean and the variance do not completely describe the index distributions presented in Table 2. Higher-order moments (e.g., skewness and kurtosis) may also differ. We focus here on the first two moments, since they are the most important determinants of the options prices considered below.

Table 2. Extended

PANEL B. Options Prices

	Rainfall Sum Index				Rainfall Deficit Index			
	— Accumulation Period —				— Accumulation Period —			
Precipitation Model	Jan–Dec	Mar–July	Apr–June	June	Jan–Dec	Mar–July	Apr–June	June
BA:								
• Put	37.04	20.56	20.15	13.82	14.95	7.10	6.31	5.16
	(6.94)	(3.95)	(3.59)	(2.26)	(2.77)	(1.30)	(1.47)	(0.95)
• Call	37.04	20.56	20.15	13.82	14.95	7.10	6.31	5.16
	(7.12)	(4.54)	(3.87)	(4.13)	(2.79)	(1.45)	(1.09)	(0.87)
IVS I:	**Beta**	**Weibull**	**Beta**	**Pearson VI**	**Beta**	**Weibull**	**Beta**	**Beta**
• Put	40.07	24.90	21.57	15.28	15.51	7.37	6.71	5.24
	(6.94)	(5.14)	(3.78)	(2.17)	(3.09)	(1.44)	(1.37)	(1.00)
• Call	37.93	22.14	20.81	21.06	15.89	7.37	7.03	5.18
	(7.19)	(4.51)	(3.71)	(6.49)	(2.70)	(1.56)	(1.25)	(1.14)
IVS II:	**Weibull**	**Erland**	**Gamma**	**Lognormal**	**Beta**	**Weibull**	**Beta**	**Beta**
• Put	43.05	20.75	19.78	15.09				
	(9.05)	(4.01)	(3.43)	(2.14)		[see IVS I]		
• Call	38.08	20.33	20.42	16.09				
	(7.25)	(4.89)	(4.20)	(4.61)				
DS I:								
• Put	28.67	20.02	15.32	11.93	3.38	4.33	3.90	3.85
	(4.40)	(3.24)	(2.98)	(2.09)	(0.82)	(1.15)	(1.20)	(1.16)
• Call	29.06	19.99	16.47	10.57	25.03	12.08	9.13	5.16
	(6.29)	(3.77)	(2.99)	(3.39)	(3.33)	(1.79)	(1.10)	(1.07)
DS II:								
• Put	31.33	21.95	17.37	12.42	6.78	6.83	5.76	4.89
	(4.85)	(3.72)	(3.50)	(2.25)	(1.35)	(1.47)	(1.28)	(1.09)
• Call	32.19	21.16	17.92	12.04	17.70	9.16	7.22	4.34
	(5.65)	(4.34)	(3.29)	(2.68)	(2.36)	(1.67)	(1.15)	(0.95)

In order to assess the reliability of the five models, panel A also displays the estimates' standard errors, which are calculated for 200 iterations using the bootstrap method.

In addition, the prices of a put and a call option on each of the eight rainfall indexes are calculated in Table 2, panel B to illustrate the consequences of statistical modeling for the valuation of weather derivatives. The options prices are calculated according to equation (1) using actual probabilities. The contract payoff W is $\max(S - I, 0) \cdot V$ for the put option and $\max(I - S, 0) \cdot V$ for the call option. The strike price S equals the sample mean of the respective indexes, and the tick size V is set to 1 € per index point. A risk-free discount rate of 5% and a maturity of nine months are assumed.

Using the empirical distribution generated by the burn analysis (BA) as a benchmark, we find from Table 2, panel A that the fit of the index value simulation and the daily simulation varies depending on the rainfall index. For example, the mean and standard deviation of the rainfall deficit from April–June and the month of June are estimated fairly well by all methods. In contrast, high deviations occur, for example, in the sum of rainfall in June.

The relative performance of the index value simulation and the daily simulation is also ambiguous. The index value simulation is superior to both variants of the daily simulation for all variants of the deficit index in the sense that its estimates are closer to the mean and the standard deviation of the empirical distribution (which themselves are random variables). In contrast, the DS II outperforms the Pearson VI distribution in the case of the rainfall sum in June, and the Weibull distribution for the accumulation period March–July. As noted above, the DS I has particular problems in estimating the volatility of the rainfall index correctly. In almost all cases, the estimator is biased downward. This problem can be mitigated by modifications of the standard model described above, but in some cases the DS II still underestimates rainfall variability, while the index value simulation tends to overestimate the volatility. This bias can be considerable—e.g., for the rainfall sum in June.

The mean and standard deviation of the index distribution are important determinants of the corresponding options prices, and hence biased estimates of these parameters translate into incorrect derivative prices. The differences between the various estimation methods are pronounced for the annual period of rainfall deficit. Fair prices for the put option calculated with the daily simulation models amount to 3.38 € and 6.78 €, respectively, while the burn analysis and the index value simulation give an option price of about 15 €. It may also happen that estimation errors in the mean and in the volatility compensate each other, i.e., the resulting option price is right for the wrong reasons. This occurs, for example, when IVS I is used to price a put option on the rainfall sum in June.

An argument sometimes presented in favor of daily simulation is that this method produces small standard errors. Actually, the daily simulation shows smaller standard errors than the burn analysis and the index value simulation in many cases of the present application, but the gain in accuracy is not as pronounced as reported by Brix, Jewson, and Ziehmann (2002) for a temperature index. It may even be that the daily simulation has slightly larger standard errors than one of the other estimation procedures. Interestingly, the estimates of the burn analysis frequently have smaller standard errors than those of the index value simulation. Contrary to this observation, the burn analysis has been criticized in the literature for producing unstable results (see, e.g., Cao and Wei, 2003; Zeng, 2000).

Estimating the Relation Between Wheat Yield and Rainfall

Estimation of the yield model is based on the yield data of a single representative farm in Brandenburg over 13 years, from 1993 to 2005. Thirteen observations seem to be a poor database for the estimation of the yield model. However, a longer time series is not available for the new federal states in Germany in general and Brandenburg in particular, since production took place under totally different conditions prior to German reunification. Hence, yield data before and after 1990 cannot be pooled.[6]

Several functional forms for the yield model have been tested—in particular, a quadratic, a logarithmic, and a linear-limitational (Leontief) production function. The latter provided the best fit in terms of R^2 for the observed data and both indexes.[7] Another advantage of the Leontief production function is that it duplicates the payoff structure of a (plain vanilla) option. Specifically, if the yield can be modeled by a linear limitation function of the weather index, then an option on this index can be constructed which exactly offsets the revenues from production (if basis risk is nonexistent).

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[6] This problem does not exist for regions in West Germany, but it is not possible to link a yield model that is calibrated for a region in West Germany with a rainfall model for Brandenburg.

[7] This finding cannot be generalized. Zhang (2003), for example, finds a quadratic production function more suitable for describing the relationship between wheat and rainfall. Vedenov and Barnett (2004) conclude that the appropriate weather-yield model varies by crop and region.

A linear-limitational production function is given by:

$$(19)\quad Q_t(I_t) = \begin{cases} d_0 + d_1 \cdot I_t + \varepsilon_t & \text{if } I_t < d_2, \\ d_3 + \varepsilon_t & \text{otherwise,} \end{cases}$$

with $t = 1, 2, \ldots, 13.$

Here, d_0, d_1, d_2, and d_3 are parameters, and ε_t is the error term of the production function. The variable I_t represents, on one hand, the rainfall sum in June, and on the other hand the rainfall deficit in the period from April 1–June 30 (with $s = 5$ and $y^{min} = 6.4$ mm).

Table 3 presents the estimated parameters of the production function. The associated test statistics show that the fit of the production function is much better for the rainfall deficit than for the rainfall sum. The magnitude of the production function estimation errors will affect the hedging effectiveness of the rainfall insurance, and therefore it is important to understand their occurrence. The rather low fit of the yield models can be partly attributed to the shortness of the yield time series. Moreover, the residuals of the regression include both production risk and geographical basis risk, since the considered farm site is located at a distance of about 40 kilometers from the weather station.

Analysis of the Hedging Effectiveness of Two Rainfall Options

At this point, we have specified all model components necessary to investigate the hedging effectiveness as described earlier. We consider two put options: the first option refers to the rainfall sum in June, while the rainfall deficit in the period from April 1–June 30 underlies the second option. These particular specifications were chosen because the correlations between the indexes and the wheat yield are then maximized. The strike levels for the two options are 130 mm and −33.7 mm, and the corresponding values for the tick size amount to 1.9 €/index point and 8.1 €/index point. Again, these parameters are determined in such a way that the hedging effectiveness of the two derivatives is maximized.

The stochastic values of the rainfall indexes are generated by means of the index value simulation (lognormal distribution for the rainfall sum and Beta distribution for the rainfall deficit) and the modified daily simulation by using 50,000 repetitions. The basis risk that is inherent to the two options is captured by the normally distributed stochastic component of the estimated production functions. When simulating the stochastic revenues according to equation (4), a constant product price of 10 €/dt is assumed.

Table 4 presents selected parameters of the revenue distribution functions with and without rain insurance, as well as their certainty equivalents. The latter refer to the negative exponential utility function with an absolute risk-aversion parameter $\lambda_a = 0.01$.[8] Note that the expected value of the profit distributions does not change when buying the option, because the option price is calculated as the expected value of the payoffs with respect to the actual probabilities.

As explained earlier, this procedure implies a weather risk market price of zero and can be justified if the rainfall index and the market portfolio are uncorrelated. In the case of a positive correlation, the expected value of the weather index at expiration should be corrected downward, thus yielding a higher price for the put options, i.e., the cost of insurance would be higher. Moreover, sellers (insurance companies, banks) will presumably charge additional premiums to cover their transaction costs. Hence, actual prices are expected to be higher than the prices reported in Table 4. This leads to a downward shift of the revenue distributions with insurance.

[8] This choice of λ_a implies rather strong risk aversion (cf. Benitez et al., 2006).

Table 3. Estimation of Production Functions and Associated Test Statistics

Parameters	Estimation	t-Value	Critical Value of t (95%)	R^2	F-Value	Critical Value of F (95%)
Rainfall Sum Index:						
d_0	52.0	7.22				
d_1	0.19	1.83		0.22	2.82	
d_2	130.0	—				
d_3	77.0	3.48	1.81			4.96
Rainfall Deficit Index:						
d_0	103.0	13.01				
d_1	0.81	4.97		0.52	10.73	
d_2	-33.7	—				
d_3	75.5	2.34				

Table 4. Hedging Effectiveness

Precipitation Model	Put Option	Option Price	Parameters of the Revenue Distribution: Mean	Standard Deviation	5%	95%	Certainty Equivalent
Rainfall Sum Index:							
IVS	Without		624.13	117.89	434.39	821.79	548.06
	With	117.21	624.13	99.79	459.96	788.24	561.59
DS II	Without		626.62	114.06	437.94	815.66	552.56
	With	113.29	626.62	100.11	461.24	792.07	564.03
Rainfall Deficit Index:							
IVS	Without		619.76	119.36	411.00	803.47	523.83
	With	107.11	619.76	78.18	491.15	748.30	574.42
DS II	Without		624.02	115.82	424.61	805.73	546.55
	With	103.73	624.02	78.37	494.93	753.13	576.25

Notes: IVS = index value simulation, DS = daily simulation; all values are in €.

In contrast to other applications (e.g., Stoppa and Hess, 2003), risk reduction using a put option on the rainfall sum is limited here due to high basis risk. The volatility of revenues with insurance is only about 15% smaller than without insurance, and the certainty equivalent increases by 2%. This result is not surprising considering the small correlation between the rainfall sum and the wheat yield. The hedging effectiveness is higher for the rainfall deficit and amounts to one-third in terms of a reduction of the standard deviation; this finding emphasizes the importance of defining an appropriate weather index. As demonstrated below, the effect of rainfall insurance may be dampened or amplified the farther or closer a farm is located in relation to the weather station.

From a methodical viewpoint, it is interesting to compare the two valuation methods. Table 4 shows that the revenue distributions of the index value simulation and the (modified) daily simulation do not differ significantly. Additionally, both approaches calculate nearly the same hedging effectiveness. The largest difference amounts to a 4% variation for the certainty equivalent in the case of the deficit index. This is remarkable since Table 2 showed considerable differences in the estimation of the mean and the standard deviation of the index distributions, in particular for the rainfall sum in June. The reason for this is that the index distributions of the rainfall sum generated by the index value simulation and the daily simulation differ mainly in the right tail, though these differences are aligned by the constant segment of the linear-limitational production function. However, this finding does not mean that the choice of modeling approach is unimportant. Other situations may exist where differences in the estimated rainfall distribution are translated more directly into the revenue distribution.

Analysis of Geographical Basis Risk

The existing empirical literature is equivocal with regard to the hedging effectiveness of weather derivatives in agriculture (see, e.g., Chen and Roberts, 2004; Edwards and Simmons, 2004; Fleege et al., 2004; Schmitz et al., 2004; Vedenov and Barnett, 2004; Manfredo and Richards, 2005). This is not surprising since the hedging effectiveness depends on several factors, which vary between applications. First, the correlation between the weather index and the yield is important. The correlation itself depends on the definition of the index and the considered product. Second, the quantification of the weather-yield relationship is subject to estimation errors, which can be pronounced. Third, the geographical basis risk has to be taken into account.

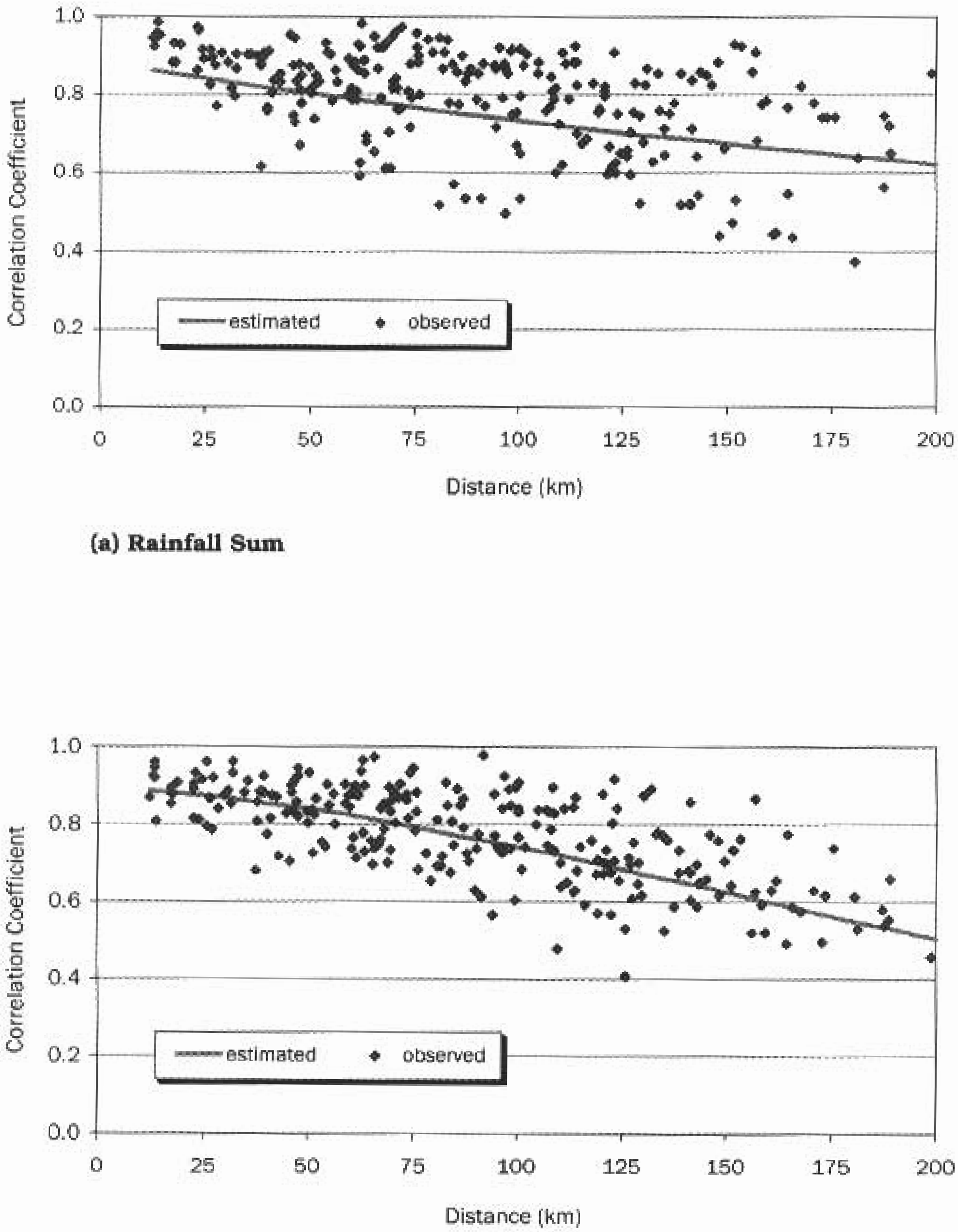
The first two issues have been discussed. Now we attempt to assess the magnitude of geographical basis risk that is inherent to the considered rainfall options by means of a de-correlation analysis. Rubel (1996) proposes the following nonlinear de-correlation function for the spatial relationship of precipitation in Europe:

(20) $\rho(d) = e_1 \cdot \exp\left(-e_2 \cdot d^{e_3}\right)$,

where $\rho(d)$ denotes the correlation coefficient between precipitation at different places, and d is the distance between the weather station and the farmer's production site. The variables e_1, e_2, and e_3 are parameters to be estimated. In spite of the de-correlation analysis being a popular instrument in meteorology, two points should be considered critically. First, the de-correlation function is invariant regarding direction. Thus, topographical differences potentially influencing precipitation are neglected. Second, Embrechts, McNeil, and Straumann (1999) identify problems of using correlation coefficients when the underlying distributions are not elliptical. Despite these weaknesses, de-correlation analysis is used in this study.

Calculation of geographical basis risk is carried out for both considered rainfall indexes (i.e., rainfall sum in June and rainfall deficit from April 1–June 30). Rainfall records for 23 weather stations in the Berlin and Brandenburg region are available between January 1983 and December 2003. For each weather station, a time series of the two rainfall indexes is calculated, and pairwise correlation coefficients are determined. Next, the distances between the stations are measured and entered into the nonlinear regression function (20). Parameter estimates for the de-correlation function are: $e_1 = 0.94$, $e_2 = 0.0033$, and $e_3 = 0.88$ for the rainfall sum, and $e_1 = 0.89$, $e_2 = 0.0001$, and $e_3 = 1.63$ for the rainfall deficit.

Figure 2 shows the graphs of the de-correlation functions which, as expected, have a negative slope. The R^2 for the rainfall sum is 0.24 and for the rainfall deficit 0.63. Therefore, the estimated de-correlation function is a better approximation to the empirical correlations in the case of the rainfall deficit. Moreover, the scatter plot reveals heteroskedasticity, i.e., the relationship between distance and correlation becomes less precise with increasing distance.



(a) Rainfall Sum

(b) Rainfall Deficit

Figure 2. De-correlation Function for Precipitation in Brandenburg, Germany

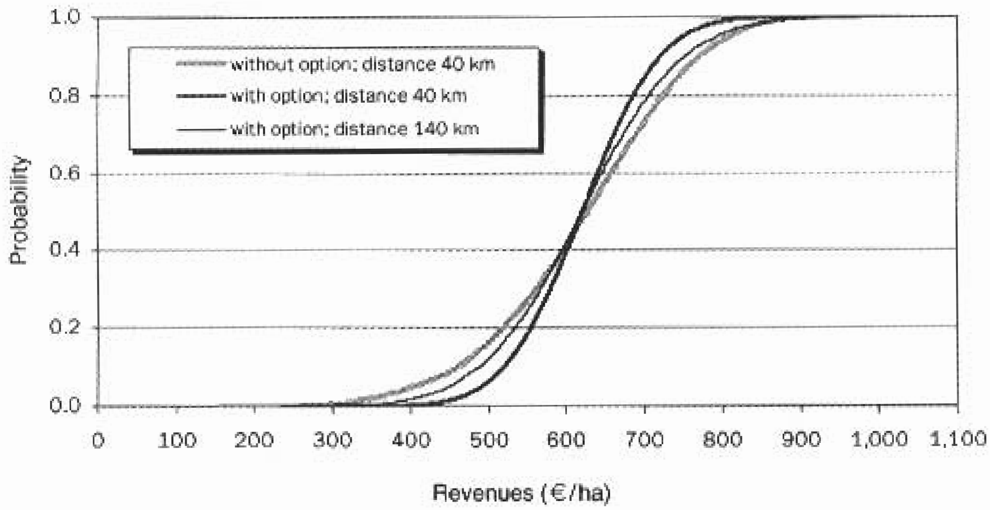


Figure 3. Revenue Distributions With and Without Hedging

From Figure 2 it can be seen that the correlation between Berlin-Tempelhof and a weather station 40 km away is approximately 0.87 for the rainfall sum and 0.86 for the rainfall deficit. These values decrease to about 0.65 and 0.5, respectively, at a distance of 200 km. This decrease is very moderate and related to the particular topographical situation in Brandenburg.[9]

Finally, we investigate the relationship between geographical basis risk and hedging effectiveness. This relationship is exemplified for the put option on the rainfall deficit index from April 1–June 30 using the index value simulation. The setting of the calculation is as before, but a second farm, which is located at a distance of 140 km from the reference weather station instead of 40 km, is also considered. According to the estimated de-correlation function, the correlation of the rainfall deficit index at these two farm locations is 0.74.

Figure 3 shows that the hedging effectiveness of a put option is considerably reduced the farther away from the reference weather station the producer is located. The standard deviation of the revenues is 78.18 at a distance of 40 km and 103.38 at a distance of 140 km. That is, more than half of the risk-reducing potential of the put option vanishes if the distance between the farm location and the reference weather station increases by 100 km.

Conclusions

In this paper we investigate three statistical approaches that can be used for modeling rain risk and pricing rain insurance. Our main interest lies with the question of whether a daily precipitation model can improve the estimation of rainfall indexes (and thereby the valuation of an index-based insurance) using simpler approaches such as burn analysis and index value simulation for comparison. Our results indicate that clear differences in the estimation results may occur among the three approaches. This finding underscores the importance of the model choice. However, it is difficult to draw an unequivocal conclusion regarding the superiority of a specific valuation approach.

[9] Salson and Garcia-Bartual (2003) report a correlation of only 0.3 at a distance of 10 km in a Mediterranean region. Paulson and Hart (2006) estimate a correlation coefficient of approximately 0.6 for one degree of latitude (110 km) in Iowa, USA. East (2005) finds a de-correlation similar to ours for southeast Australia.

Insofar as our results do not confirm others' previous experience in the context of temperature modeling, daily simulation is generally preferred. On the one hand, applying daily simulation has the advantage of yielding smaller confidence intervals for the resulting indexes and prices compared with the nonparametric burn analysis and the index value simulation, though this advantage seems to be much smaller for rainfall than for temperature. On the other hand, the danger of a rather sophisticated daily precipitation model being wrongly specified is relatively high; such a risk is precluded when the precipitation index is estimated directly.

In the present application, the daily simulation model tends to underestimate the volatility of monthly rainfall. This pitfall may be of minor importance in the context of meteorological or hydrological applications, but it is severe when the model is used for risk assessment and derivative pricing. Some measures to reduce this bias have been discussed and successfully implemented in this paper. Nevertheless, the problem deserves further attention.

Another shortcoming of the presented daily rainfall model is the ignorance of long-term (interannual) variability of the parameters. This means that trends, or an increase of rainfall volatility due to climatic changes, are not captured by this model. In principle, however, it is also possible to incorporate interannual variability into daily precipitation models (cf. Wilks and Wilby, 1999). We conclude that the preferential statistical approach to weather derivative pricing depends on the context of its application. Ideally, more than one model should be implemented, and differences of the models' outcomes should be carefully analyzed. A systematic model validation based on quasi-ex ante forecasts is suggested as a subject for further research.

Regardless of the issue of the appropriate statistical method, the following practical conclusions can be drawn. The risk-reducing effect of precipitation derivatives is much more regionally confined than is the case with temperature-related derivatives. In the example of Brandenburg considered here, the correlation between the precipitation index of the weather station Berlin-Tempelhof and a remote farm site decreases to a value of 0.75 at a distance of 100 km. If one additionally takes into account the stochastic relation between precipitation and production, the use of rainfall derivatives as risk management tools in agriculture appears questionable, at least for conditions comparable to those in this study.

It follows that potential suppliers of rainfall insurance should introduce a dense network of weather stations as reference points for the rainfall index in order to increase the attractiveness of this type of insurance, although this may lead to fragmented demand. Moreover, the specification of adequate weather indexes also requires further study. Simple rainfall indexes, upon which we focus here, may not be specific enough from the viewpoint of many producers. The inclusion of additional weather variables in the weather index (e.g., temperature or humidity) could help mitigate the problem of basis risk.

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Appendix

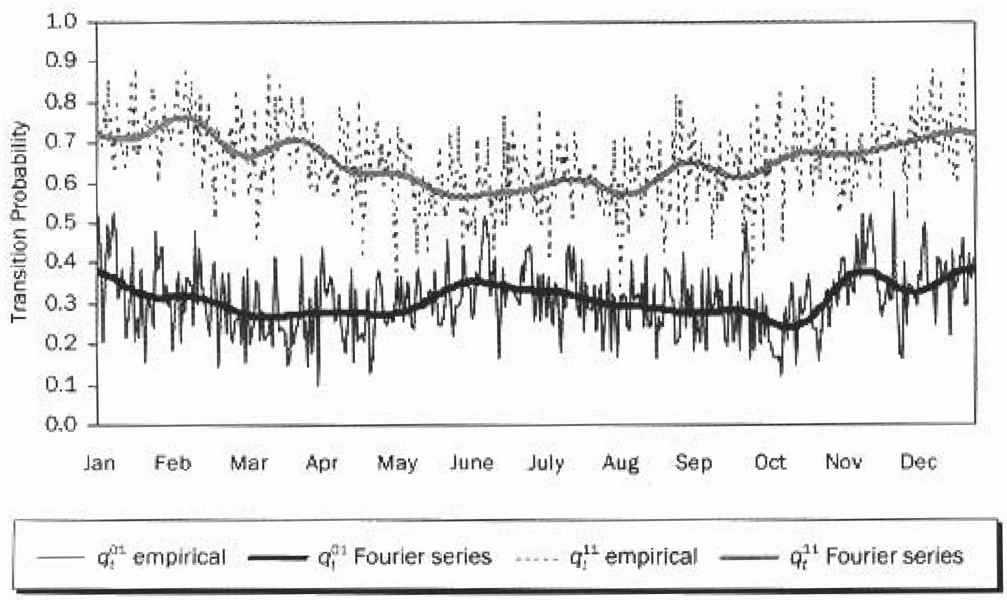
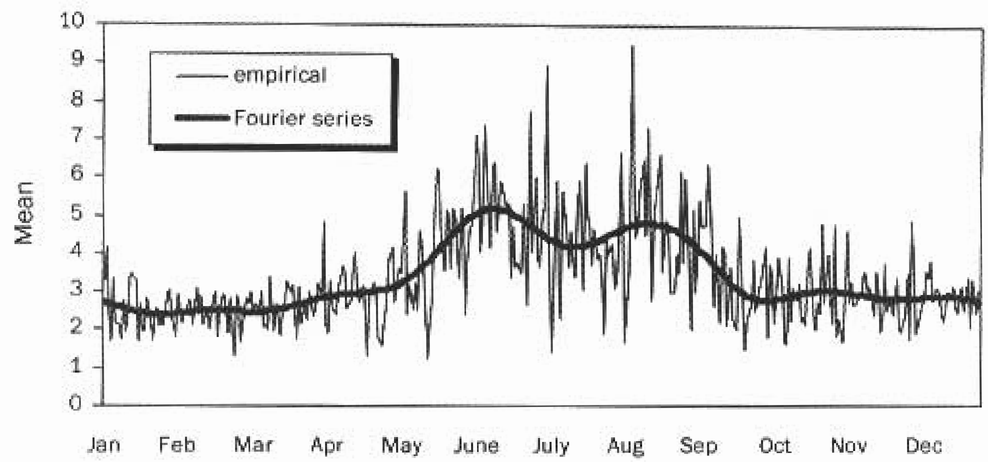
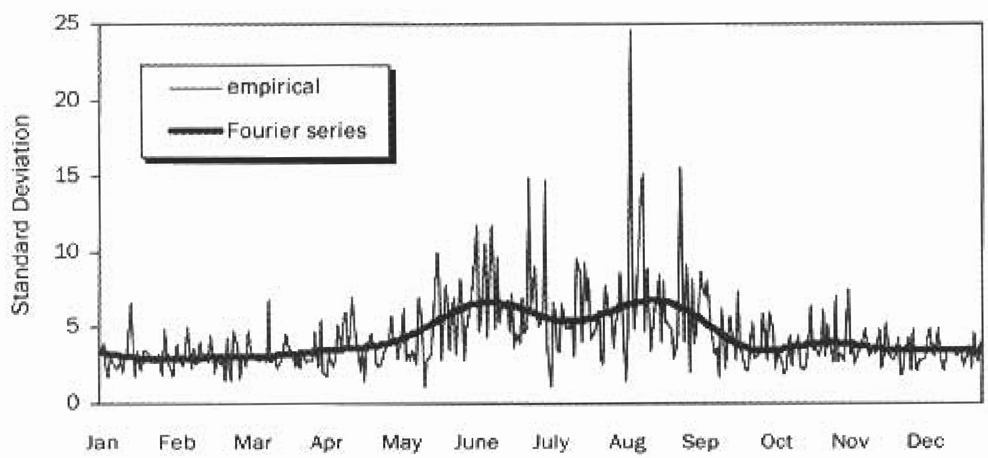


Figure A1. Conditional Transition Probabilities



(a) Mean



(b) Standard Deviation

Figure A2. Conditional Mean and Standard Deviation of Daily Precipitation